

# POLARIZED MUON DECAY AT REST WITH V+A INTERACTION

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In this paper, we analyze the polarized muon decay at rest (PMDaR) and elastic neutrino-electron scattering (ENES) admitting the non-standard V+A interaction in addition to standard V-A interaction. Considerations are made for Dirac massive muon neutrino and electron antineutrino. Moreover, muon neutrinos are transversely polarized. It means that the outgoing muon-neutrino beam is a mixture of the left- and right-chirality muon neutrinos and has a fixed direction of transverse spin polarization with respect to production plane. We show that the angle-energy distribution of muon neutrinos contains the interference terms between the standard V-A and exotic V+A couplings, which are proportional to the transverse components of muon neutrino spin polarization. They do not vanish in a limit of massless neutrino and include the relative phases to test the CP violation. In consequence, it allows to calculate a neutrino flux and an expected event number in the ENES (detection process) both for the standard model prediction and the case of neutrino left-right mixture.

PACS numbers: 13.35.Bv, 13.15.+g, 14.60.St

## 1. Introduction

Polarized muon decay at rest (PMDaR) is the appropriate laboratory to test both Lorentz structure and violation of combined symmetry CP of the purely leptonic charged weak interactions [1, 2]. In addition, the above process may be used to probe the question of lepton number violation [3]. The various low energy precise measurements of muon decay such as; the spectral shape, angular distribution, and polarization of the outgoing electrons (positrons) led among other to a vector-axial (V-A) structure [4] of the electroweak interactions of Standard Model (SM)[5, 6, 7]. This means that only left-chirality Dirac neutrinos may take part in the charged and neutral current weak interaction. The V-A coupling also leads to maximal parity violation and massless neutrinos. Although the SM agrees with all data up to presently available energies, it contains the large number of undetermined parameters, so the existence of new physics is expected. One of the fundamental aspects, which are not explained in the SM, is the origin of parity violation at current energies. In addition, the experimental precision still does not rule out the participation of the exotic weak interactions with right-chirality neutrinos. This situation led to the formulation of various non-standard gauge models, which have the  $SU(2)_L \times U(1)$  as own subgroup and their predictions agree with the SM at low energies. Many theoretical schemes going beyond the SM include new interactions whose structure is different from V-A. There is a rich literature concerning non-standard models. We mean here the various versions of left-right symmetric models (LRSM) with the  $SU(2)_L \times SU(2)_R \times U(1)$  as the gauge group [8] and the models including mechanisms which admit scalar, tensor and pseudoscalar structures. The LRSM emerged first in the framework of grand unified theories (GUT). They restore the parity symmetry at high energies and give its violation at low energies as a result of gauge symmetry breaking.

The mentioned above muon decay is still one of the basic processes used in searching for effects connected with the  $V + A$  weak interaction at low energies and possibility of CP violation. There are many theoretical and experimental papers concerning these aspects [9, 10, 11, 12, 13]. At present, there is no experimental evidence for the deviations from the SM for the Michel parameters [14]. These parameters contain only contributions from the right-chirality neutrinos in the form of sums of the squares of certain combinations of the coupling constants. It seems meaningful to search for new observables with the linear contributions from the right-chirality couplings. The proper candidates can be neutrino observables, including an information on the transverse components of neutrino spin polarization, both T-even and T-odd. Today, direct measurements of such observables in the PMDaR are impossible, so a reasonable solution seems to be the scat-

tering of polarized neutrino beam coming from the PMDaR off electrons of target. A signature of existence of the right-chirality neutrinos would be detection of deviation from the SM prediction in the recoil electron angular distribution. However, high-precision measurements of the neutrino observables require large detectors and strong polarized (anti)neutrino sources, and long time of experiment duration (one year and longer). Moreover, the detectors should measure both polar angle and azimuthal angle of the outgoing electron momentum with a high resolution, and must also distinguish the electrons from various potential background sources (e.g., the electron produced by neutrino-nucleon scattering can give a final state that is often consistent with a single recoil electron coming from neutrino-electron scattering). New types of detectors with a very low threshold  $\propto 10\text{ eV}$  are intensively developed by many groups. We mean the silicon cryogenic detectors based on the ionization-into-heat conversion effect and the high purity germanium detectors with the internal amplification of a signal in the electric field [15]. It is also worth mentioning that TRIUMF Collaboration carried out the precise measurement with the decay of polarized muons at rest. In order to minimize depolarization effects the muons were stopped in the pure metal foil and liquid-He targets [1].

In this paper, we focus on the study of the PMDaR (production process of polarized muon-neutrino beam) and of the ENES (detection process of exotic effects) in the presence of  $V+A$  interaction, assuming Dirac massive (anti)neutrinos. Our analysis is not made in the framework of concrete version of the left-right symmetric model. The main goal is to show how the angle-energy distribution of the muon neutrinos produced in the PMDaR depends on the interference terms between the standard vector coupling of the left-chirality neutrinos and exotic vector couplings of the right-chirality ones. It will allow to calculate the flux of muon neutrinos, both for the SM prediction and the case of neutrino left-right mixture. Next, we will calculate the expected number of events in the ENES, when the incoming neutrino beam comes from the PMDaR and is transversely polarized. The neutrino flux and event number are found for a detector in the shape of flat circular ring with a low threshold.

This paper is organized as follows: Sec. II contains a basic assumptions as to the production process of muon-neutrino beam. In the sec. III, the results for the energy-angle distribution of muon neutrinos coming from the PMDaR are presented. In the sec. IV we present the numerical results concerning the neutrino flux and expected number of events both for the SM with only left-chirality muon neutrinos and the case of left-right chirality muon-neutrino mixture, when muon-neutrino beam is transversely polarized. Finally, we summarize our considerations.

The results are presented in a limit of infinitesimally small mass for all

Coupling constants	SM	Current limits
$ g_{LL}^V $	1	$> 0.960$
$ g_{LR}^V $	0	$< 0.036$
$ g_{RL}^V $	0	$< 0.104$
$ g_{RR}^V $	0	$< 0.034$

Table 1. Current limits on the non-standard couplings

the particles produced in the muon decay. The density operators [16] for the polarized initial muon and for the polarized outgoing muon neutrino are used, see Appendix. We use the system of natural units with  $\hbar = c = 1$ , Dirac-Pauli representation of the  $\gamma$ -matrices and the  $(+, -, -, -)$  metric [17].

## 2. Polarized muon decay at rest - basic assumptions

We assume that the polarized muon decay at rest ( $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ ) is a source of the muon neutrino beam. This process is described at a level of lepton-number-conserving four-fermion point interaction. We admit a presence of the exotic vector  $g_{LR,RL,RR}^V$  couplings in addition to the standard vector  $g_{LL}^V$  coupling. It means that the outgoing muon neutrino flux is a mixture of the left-chirality and right-chirality muon neutrinos. The amplitude for the above process is of the form:

$$\begin{aligned}
M_{\mu^-} = & \frac{G_F}{\sqrt{2}} \{ g_{LL}^V (\bar{u}_e \gamma_\alpha (1 - \gamma_5) v_{\nu_e}) (\bar{u}_{\nu_\mu} \gamma^\alpha (1 - \gamma_5) u_\mu) \\
& + g_{RR}^V (\bar{u}_e \gamma_\alpha (1 + \gamma_5) v_{\nu_e}) (\bar{u}_{\nu_\mu} \gamma^\alpha (1 + \gamma_5) u_\mu) \\
& + g_{LR}^V (\bar{u}_e \gamma_\alpha (1 - \gamma_5) v_{\nu_e}) (\bar{u}_{\nu_\mu} \gamma^\alpha (1 + \gamma_5) u_\mu) \\
& + g_{RL}^V (\bar{u}_e \gamma_\alpha (1 + \gamma_5) v_{\nu_e}) (\bar{u}_{\nu_\mu} \gamma^\alpha (1 - \gamma_5) u_\mu), \}
\end{aligned} \tag{1}$$

where  $v_{\nu_e}$  and  $\bar{u}_e$  ( $u_\mu$  and  $\bar{u}_{\nu_\mu}$ ) are the Dirac bispinors of the outgoing electron antineutrino and electron (initial muon and final muon neutrino), respectively.  $G_F = 1.16639(1) \times 10^{-5} \text{ GeV}^{-2}$  [14] is the Fermi constant. The coupling constants are denoted as  $g_{LL}^V$  and  $g_{LR,RL,RR}^V$  respectively to the chirality of the final electron and initial stopped muon. Our analysis is carried out in the limit of massless (anti)neutrino, then left-chirality muon neutrino possesses negative helicity, while the right-chirality one has positive helicity, see [13]. In the SM, only  $g_{LL}^V$  is non-zero value. The table 1 displays the current limits for the  $g_{LL}^V$ ,  $g_{LR,RL,RR}^V$  couplings. Because we allow for the non-conservation of the combined symmetry CP, all the coupling constants

$g_{LL}^V, g_{LR,RL,RR}^V$  are complex.

The initial muon is at rest and polarized. The unit vector in the LAB system  $\hat{\eta}_\mu$  denotes the muon polarization for a single muon decay. The production plane is spanned by the direction of the muon polarization  $\hat{\eta}_\mu$  and of the outgoing muon neutrino LAB momentum unit vector  $\hat{\mathbf{q}}$ . As is well known, in this plane, the polarization vector  $\hat{\eta}_\mu$  can be expressed, with respect to the  $\hat{\mathbf{q}}$ , as a sum of the longitudinal component of the muon polarization  $(\hat{\eta}_\mu \cdot \hat{\mathbf{q}})\hat{\mathbf{q}}$  and transverse component of the muon polarization  $\eta_\mu^\perp$ , which is defined as  $\eta_\mu^\perp = \hat{\eta}_\mu - (\hat{\eta}_\mu \cdot \hat{\mathbf{q}})\hat{\mathbf{q}}$ .

### 3. Energy-angle distribution of muon neutrinos

In this section, we show how the energy-angle distribution of muon neutrinos depends on the interference terms between the standard and exotic couplings in the limit of vanishing electron-antineutrino and muon-neutrino masses.

The proper formula, obtained after the integration over all the momentum directions of the outgoing electron and electron antineutrino, is of the form:

$$\frac{d^2\Gamma}{dyd\Omega_\nu} = \left( \frac{d^2\Gamma}{dyd\Omega_\nu} \right)_{(LL+LR+RL+RR)} + \left( \frac{d^2\Gamma}{dyd\Omega_\nu} \right)_{(INT)} \quad (2)$$

$$\begin{aligned} \left( \frac{d^2\Gamma}{dyd\Omega_\nu} \right)_{(LL+LR+RL+RR)} &= \frac{G_F^2 m_\mu^5}{768\pi^4} \left[ (1 - \hat{\eta}_\nu \cdot \hat{\mathbf{q}})(|g_{LL}^V|^2 + |g_{RL}^V|^2) \right. \\ &\quad \cdot y^2(-2y + 3 - (2y - 1)\hat{\eta}_\mu \cdot \hat{\mathbf{q}}) + (1 + \hat{\eta}_\nu \cdot \hat{\mathbf{q}})(|g_{LR}^V|^2 + |g_{RR}^V|^2) \\ &\quad \left. \cdot y^2(-2y + 3 + (2y - 1)\hat{\eta}_\mu \cdot \hat{\mathbf{q}}) \right], \end{aligned} \quad (3)$$

$$\begin{aligned} \left( \frac{d^2\Gamma}{dyd\Omega_\nu} \right)_{(INT)} &= \frac{G_F^2 m_\mu^5}{384\pi^4} y^2 \left[ \left( \text{Re}(g_{LL}^V g_{LR}^{V*}) + \text{Re}(g_{RL}^V g_{RR}^{V*}) \right) (\eta_\nu^\perp \cdot \hat{\eta}_\mu) \right. \\ &\quad \left. - \left( \text{Im}(g_{LL}^V g_{LR}^{V*}) + \text{Im}(g_{RL}^V g_{RR}^{V*}) \right) \eta_\nu^\perp \cdot (\hat{\mathbf{q}} \times \hat{\eta}_\mu) \right]. \end{aligned} \quad (4)$$

Here,  $y = \frac{2E_\nu}{m_\mu}$  is the reduced muon neutrino energy for the muon mass  $m_\mu$ , it varies from 0 to 1, and  $d\Omega_\nu$  is the solid angle differential for  $\nu_\mu$  momentum  $\hat{\mathbf{q}}$ .

Equation (4) includes the interference term between the  $g_{LL}^V$  (left-chirality  $\nu_\mu$ ) and exotic  $g_{LR}^V$  (right-chirality  $\nu_\mu$ ) couplings, so it is linear in the exotic coupling contrary to Eq. (3) and the interference between the  $g_{RL}^V$  and  $g_{RR}^V$  couplings.

By  $\hat{\boldsymbol{\eta}}_{\boldsymbol{\nu}}$ ,  $(\hat{\boldsymbol{\eta}}_{\boldsymbol{\nu}} \cdot \hat{\mathbf{q}})\hat{\mathbf{q}}$ , and  $\hat{\boldsymbol{\eta}}_{\boldsymbol{\nu}}^{\perp}$  we denote the unit polarization vector, its longitudinal component, and transverse component of the outgoing  $\nu_{\mu}$  in its rest system, respectively.

It is necessary to point out that there is the different dependence on the  $y$  between quadratic terms and interferences. For  $\hat{\boldsymbol{\eta}}_{\mu} \cdot \hat{\mathbf{q}} = 0$ , the interference part can be rewritten in the following way:

$$\left( \frac{d^2\Gamma}{dyd\Omega_{\nu}} \right)_{(INT)} = \frac{G_F^2 m_{\mu}^5}{384\pi^4} |\boldsymbol{\eta}_{\nu}^{\perp}| |\boldsymbol{\eta}_{\mu}^{\perp}| \quad (5)$$

$$\cdot \left\{ |g_{LL}^V| |g_{LR}^V| \cos(\phi - \alpha) + |g_{RL}^V| |g_{RR}^V| \cos(\phi - \beta) \right\} y^2,$$

where  $\phi$  is the angle between the direction of  $\boldsymbol{\eta}_{\nu}^{\perp}$  and the direction of  $\boldsymbol{\eta}_{\mu}^{\perp}$  only;  $\alpha \equiv \alpha_V^{LL} - \alpha_V^{LR}$ ,  $\beta \equiv \alpha_V^{RL} - \alpha_V^{RR}$  are the relative phases between the  $g_{LL}^V$ ,  $g_{LR}^V$ , and  $g_{RL}^V$ ,  $g_{RR}^V$  couplings.

It can be noticed that the relative phases  $\alpha, \beta$  different from  $0, \pi$  would indicate the CP violation in the CC weak interaction. We also see that the interference terms do not vanish in the limit of vanishing electron-antineutrino and muon-neutrino masses. This independence of the neutrino mass enables the measurement of the relative phases  $\alpha, \beta$  between these couplings. The interference part, Eq. (5), includes only the contributions from the transverse component of the initial muon polarization  $\boldsymbol{\eta}_{\mu}^{\perp}$  and the transverse component of the outgoing neutrino polarization  $\boldsymbol{\eta}_{\nu}^{\perp}$ . Both transverse components are perpendicular with respect to the  $\hat{\mathbf{q}}$ .

Using the current data [14], we calculate the upper limit on the magnitude of the transverse neutrino polarization and lower bound for the longitudinal neutrino polarization, see [13]:

$$|\boldsymbol{\eta}_{\nu}^{\perp}| = 2\sqrt{Q_L^{\nu}(1 - Q_L^{\nu})} \leq 0.103, \quad (6)$$

$$|\hat{\boldsymbol{\eta}}_{\nu} \cdot \hat{\mathbf{q}}| = |1 - 2Q_L^{\nu}| \geq 0.995, \quad (7)$$

$$Q_L^{\nu} = 1 - |g_{RR}^V|^2 - |g_{LR}^V|^2 \geq 0.997, \quad (8)$$

where  $Q_L^{\nu}$  is the probability of finding the left-chirality  $\nu_{\mu}$ .

If the neutrino beam comes from the unpolarized muon decay at rest, there is no interference terms in the spectral function of muon neutrinos.

It is worth noticing that the effects coming from the neutrino mass and mixing are very small and they may be neglected. In order to show this, we use the final density matrix for the mass states  $m_1, m_2$  of  $\nu_{mu}$  to avoid breaking the fundamental principles of Quantum Field Theory. We assume that at the neutrino detector (target)  $\nu_{\mu} = \cos\theta\nu_1 + \sin\theta\nu_2$ . In this way,

the differential neutrino spectrum is of the form:

$$\begin{aligned}\frac{d^2\Gamma}{dyd\Omega_\nu} &= \cos^2\theta \frac{d^2\Gamma}{dy_1d\Omega_\nu} + \sin^2\theta \frac{d^2\Gamma}{dy_2d\Omega_\nu} \\ &= \gamma_{(V)}^{(\phi)} \left[ y_1^2 + \sin^2\theta \frac{\delta m_\nu^2}{m_\mu^2} + O\left(\frac{\delta m_\nu^2}{m_\mu^2}\right) \right],\end{aligned}\quad (9)$$

where  $\gamma^{(\phi)} = \frac{G_F^2 m_\mu^5}{384\pi^4} |\boldsymbol{\eta}_\nu^\perp| |\boldsymbol{\eta}_\mu^\perp| \left\{ |g_{LL}^V| |g_{LR}^V| \cos(\phi - \alpha) + |g_{RL}^V| |g_{RR}^V| \cos(\phi - \beta) \right\}$ . We see that the linear contribution from the mass mixing  $\frac{\delta m_\nu^2}{m_\mu^2}$  is of the order of  $10^{-19}$ , taking into account the available data, so this effect does not affect the transverse neutrino polarization.

#### 4. Neutrino flux and number of events in neutrino-electron scattering

In order to find the neutrino flux, we assume that the hypothetical detector has the shape of flat circular ring with a low threshold  $T_e^{th} = 10 \text{ eV}$  ( $y_e^{th} = 0.0062$ ) corresponding to  $E_\nu^{min} = 1603.44 \text{ eV}$ . In addition, one assumes that neutrino source is located in the center of the ring detector and polarized perpendicular to the ring. It means that we must integrate the angle-energy distribution of muon neutrinos over the neutrino energy in the range  $[1603.44 \text{ eV}, m_\mu/2]$ , and over the  $d\Omega_\nu$  in the proper range, i.e.  $\phi_\nu \in [0, 2\pi], \theta_\nu \in [\pi/2 - \delta, \pi/2 + \delta]$ . Then, it is multiplied by  $\frac{N_\mu}{S_D}$  yet, where  $N_\mu = 10^{21}$  is the number of polarized muons decaying per one year.  $S_D = 4\pi R^2 \sin\delta$ , where  $R = L = 2205 \text{ cm}$  is the inner radius of the detector that is equal to the distance between the muon neutrino source and detector,  $\delta = 0.01$ . In this way one gets the information on the number of muon neutrinos passing through  $S_D$  in the direction perpendicular to the  $\hat{\boldsymbol{\eta}}_\mu$ . Fig. 1 shows the dependence of  $\frac{N_\mu}{S_D} \frac{d\Gamma}{dy}$  on the  $y$  for the muon neutrinos emitted perpendicular to  $\hat{\boldsymbol{\eta}}_\mu$ , when  $V+A$  interaction is admitted. The most stringent contribution comes from the left-chirality neutrinos and it is of order of  $10^{19}$ . The interference gives the contribution of order of  $10^{16}$ . Fig.2 illustrates the plot of  $\frac{N_\mu}{S_D} \frac{d\Gamma}{d(\cos\theta_\nu)}$  as a function of  $\theta_\nu$  (angle between  $\hat{\boldsymbol{\eta}}_\mu$  and  $\hat{\mathbf{q}}$ ) for L-R mixture. The interference contribution is of order of  $10^{18} \sin\theta_\nu$ . Fig. 3 displays the dependence of neutrino flux  $N_\nu^\perp$  on the  $\phi$  angle for the case of CP conservation and CP violation in presence of  $V+A$  interaction, when  $\hat{\mathbf{q}} \perp \hat{\boldsymbol{\eta}}_\mu$ . To calculate the expected event number, we assume that the detector with high efficiency ( $\epsilon(y) = 1$  at the muon neutrino energies above threshold) contains a large number of electrons in the fiducial volume ( $N_e = 2.097 \cdot 10^{34}$  corresponds to the 75 Kton of the Fe). Moreover,

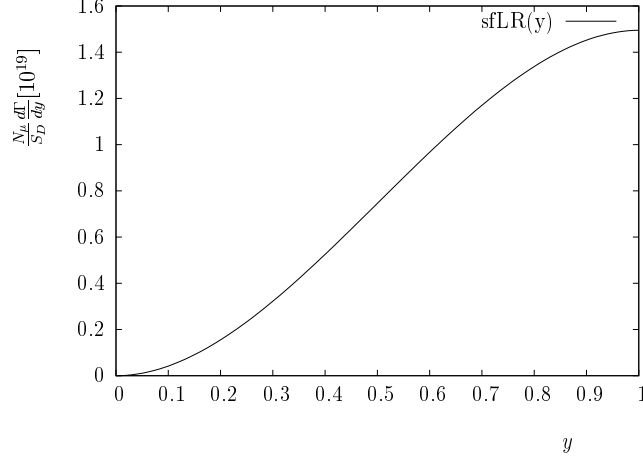


Fig.1. Plot of the  $\frac{N_\mu}{S_D} \frac{d\Gamma}{d(\cos\theta_\nu)}$  as a function of  $\theta_\nu$ , when  $V + A$  interaction is admitted.

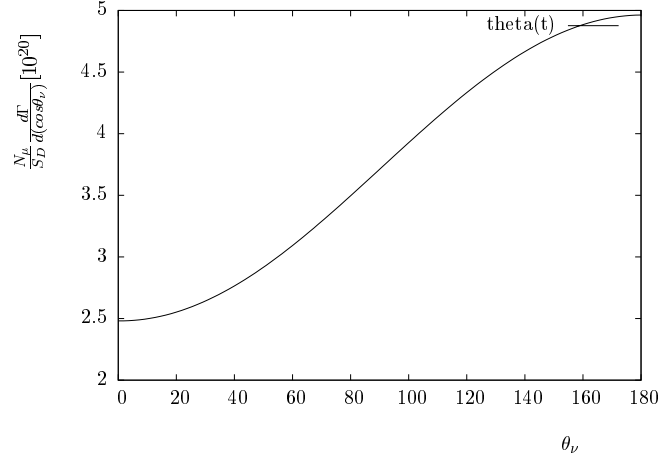


Fig.2. Plot of the  $\frac{N_\mu}{S_D} \frac{d\Gamma}{d(\cos\theta_\nu)}$  as a function of  $\theta_\nu$ , when  $V + A$  interaction is admitted.

the detector is able to measure both the polar angle and azimuthal angle of outgoing electrons with a high resolution. We also need the laboratory differential cross section for the  $\nu_\mu e^-$  scattering. An appropriate transition amplitude includes the complex coupling constants denoted as  $g_V^L, g_A^L$  and



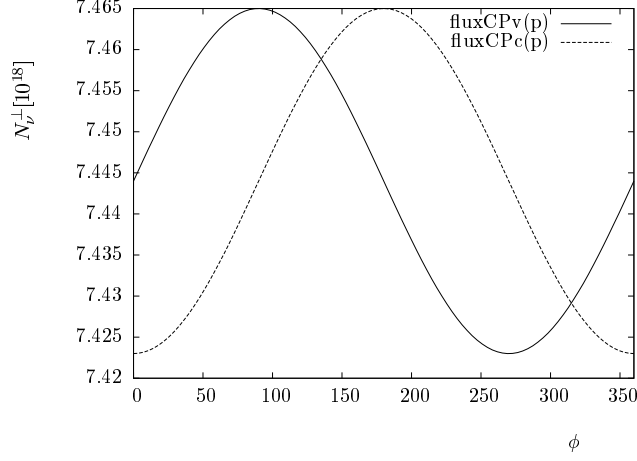


Fig. 3. Plot of the  $N_\nu^\perp$  as a function of  $\phi$ : solid line shows a case of CP violation, while short-dashed line concerns CP conservation in presence of  $V + A$  interaction, when  $\hat{\mathbf{q}} \perp \hat{\boldsymbol{\eta}}_\mu$ .

$g_V^R, g_A^R$  respectively to the initial muon neutrino of left- and right-chirality:

$$M_{\nu_\mu e} = \frac{G_F}{\sqrt{2}} \{ (\bar{u}_{e'} \gamma^\alpha (g_V^L - g_A^L \gamma_5) u_e) (\bar{u}_{\nu_{\mu'}} \gamma_\alpha (1 - \gamma_5) u_{\nu_\mu}) + (\bar{u}_{e'} \gamma^\alpha (g_V^R + g_A^R \gamma_5) u_e) (\bar{u}_{\nu_{\mu'}} \gamma_\alpha (1 + \gamma_5) u_{\nu_\mu}) \}, \quad (10)$$

where  $u_e$  and  $\bar{u}_{e'}$  ( $\bar{u}_{\nu_\mu}$  and  $u_{\nu_{\mu'}}$ ) are the Dirac bispinors of the initial and final electron (muon neutrino) respectively;  $g_V^L = -0.035 \pm 0.017$ ,  $g_A^L = -0.503 \pm 0.017$  [18];

$$\begin{aligned} g_V^L - g_A^L \gamma_5 &= \frac{g_V^L - g_A^L}{2} (1 + \gamma_5) + \frac{g_V^L + g_A^L}{2} (1 - \gamma_5), \\ g_V^R + g_A^R \gamma_5 &= \frac{g_V^R + g_A^R}{2} (1 + \gamma_5) + \frac{g_V^R - g_A^R}{2} (1 - \gamma_5). \end{aligned} \quad (11)$$

The obtained formula has the form:

$$\frac{d^2 \sigma}{dy_e d\phi_e} = \left( \frac{d^2 \sigma}{dy_e d\phi_e} \right)_{(V-A)} + \left( \frac{d^2 \sigma}{dy_e d\phi_e} \right)_{(V+A)}, \quad (12)$$

$$\begin{aligned} \left( \frac{d^2 \sigma}{dy_e d\phi_e} \right)_{(V-A)} &= B \left\{ (1 - \hat{\boldsymbol{\eta}}_\mu \cdot \hat{\mathbf{q}}) \left[ (g_V^L + g_A^L)^2 + (g_V^L - g_A^L)^2 (1 - y_e)^2 - \frac{m_e y_e}{E_\nu} ((g_V^L)^2 - (g_A^L)^2) \right] \right\}, \end{aligned} \quad (13)$$

Case	Neutrino flux $\Phi_\nu^\perp [cm^{-2}s^{-1}]$	Event number $N_e$
1. SM	$7.483 \cdot 10^{18}$	$8.854 \cdot 10^9$
2. $g_{LL}^V + g_{RL}^V + g_{LR}^V + g_{RR}^V$	$7.444 \cdot 10^{18}$	$9.854 \cdot 10^9$
3. $g_{LL}^V g_{LR}^V + g_{RL}^V g_{RR}^V$	$2.079 \cdot 10^{16}$	$2.95 \cdot 10^7$
4.2 + 3	$7.465 \cdot 10^{18}$	$9.884 \cdot 10^9$

Table 2. The values of neutrino flux and event number predicted per one year for the SM, and in the presence of  $V + A$  interaction.

$$\left( \frac{d^2\sigma}{dy_e d\phi_e} \right)_{(V+A)} = B \left\{ (1 + \hat{\boldsymbol{\nu}} \cdot \hat{\mathbf{q}}) \left[ (g_V^R + g_A^R)^2 + (g_V^R - g_A^R)^2 (1 - y_e)^2 - \frac{m_e y_e}{E_\nu} \left( (g_V^R)^2 - (g_A^R)^2 \right) \right] \right\}, \quad (14)$$

where  $y_e \equiv \frac{T_e}{E_\nu} = \frac{m_e}{E_\nu} \frac{2\cos^2\theta_e}{(1 + \frac{m_e}{E_\nu})^2 - \cos^2\theta_e}$  is the ratio of the kinetic energy of the recoil electron  $T_e$  to the incoming neutrino energy  $E_\nu$ ;  $B \equiv \frac{E_\nu m_e}{4\pi^2} \frac{G_F^2}{2}$ ;  $\theta_e$  is the angle between the direction of the outgoing electron momentum  $\hat{\mathbf{p}}_e$  and the direction of the incoming neutrino momentum  $\hat{\mathbf{q}}$  (recoil electron scattering angle);  $m_e$  is the electron mass;  $\phi_e$  is the angle between the production plane and the reaction plane spanned by the  $\hat{\mathbf{p}}_e$  and  $\hat{\mathbf{q}}$ .

The above formula describes the scattering of transversely polarized muon-neutrino beam on the unpolarized electrons in the limit of vanishing neutrino mass. In our case the incoming neutrino beam is the mixture of the left-chirality neutrinos detected in the standard  $g_V^L, g_A^L$  weak interactions and right-chirality ones detected in the exotic  $g_V^R, g_A^R$  weak interactions. We see that all the interference terms between the standard and exotic couplings vanish in the massless neutrino limit. There are only squared contributions from exotic couplings proportional to the the longitudinal neutrino polarization. It is necessary to point out that there is an observable containing the linear contributions from the exotic couplings which are independent of the neutrino mass. We mean the angular distribution of outgoing neutrinos, but such a measurement is totally unrealistic.

The number of events is found as result of integration of the differential cross section over the  $T_e$  in the range  $[T_e^{th} = 10 \text{ eV}, T_e(E_\nu)]$ . So, the received expression together with the spectral function integrates over the  $E_\nu$  in the range  $[1603.44 \text{ eV}, m_\mu/2]$ . Finally, we must multiply it by  $N_\mu/S_D$ .

Using the available data [14, 18, 19], we get the flux of neutrino beam and the number of events both for the SM and the case of left-right mixture, see table 2.

## 5. Conclusions

In this paper, we investigated the PMDaR and ENES in the presence of the exotic  $V + A$  interaction, when the (anti)neutrinos have Dirac nature and are massive.

We have shown that the angle-energy distribution of muon neutrinos produced in the PMDaR includes the terms with interference between the  $g_{LL}^V$  (left-chirality  $\nu_\mu$ ) and exotic  $g_{LR}^V$  (right-chirality  $\nu_\mu$ ) couplings, which are independent of the muon neutrino and electron antineutrino masses. These interferences are exclusively proportional to the T-even and T-odd transverse components of neutrino polarization  $\eta_\mu^\perp$ .

Next, we have calculated the angular distribution of recoil electrons for the ENES, when the incoming polarized neutrino beam comes from the PMDaR. We have demonstrated that the laboratory differential cross section contains only the squared contributions from the exotic couplings, because all the interferences are suppressed by the neutrino mass. It means that the angular distribution should be azimuthally symmetric. It is worth noting that the SM also predicts the similar regularity for the angular distribution. Using the current data, we have computed the flux of muon neutrinos and expected event number for a given configuration of detector, both for the SM prediction and for the case of L-R neutrino mixture.

Unfortunately, the detection of new effects at present experimental precision would be very difficult, even if one had the strong neutrino source ( $10^{21}$  or more polarized muons decaying at rest per year) and the high-precision large detector ( $10^{34}$  or more target-electrons) with the low threshold, measuring both polar angle and azimuthal angle of the outgoing electron momentum. We have also displayed that the eventual effects connected with the neutrino mass and mixing in the spectral function are totally inessential ( $\sim 10^{-19}$ ). We plan to search for the other polarized (anti)neutrino beams, which could be interesting from the aspect of observable effects caused by the exotic right-chirality states. We expect some interest of the neutrino laboratories working with polarized muon decay and artificial (anti)neutrino sources, and neutrino beams, e.g. KARMEN, PSI, TRIUMF, BooNE, SUPERKAMIOKANDE.

## Appendix A

### *Appendix: Four-vector of antineutrino spin polarization and density operator*

The formula for the spin polarization 4-vector of massive neutrino  $S'$  moving with the momentum  $\mathbf{q}$  is as follows:

$$S' = (S'^0, \mathbf{S}'), \quad (\text{A.1})$$

$$S'^0 = \frac{|\mathbf{q}|}{m_\nu} (\hat{\boldsymbol{\eta}}_\nu \cdot \hat{\mathbf{q}}), \quad (\text{A.2})$$

$$\mathbf{S}' = \left( \frac{E_\nu}{m_\nu} (\hat{\boldsymbol{\eta}}_\nu \cdot \hat{\mathbf{q}}) \hat{\mathbf{q}} + \hat{\boldsymbol{\eta}}_\nu - (\hat{\boldsymbol{\eta}}_\nu \cdot \hat{\mathbf{q}}) \hat{\mathbf{q}} \right), \quad (\text{A.3})$$

where  $\hat{\boldsymbol{\eta}}_\nu$  is the unit 3-vector of the neutrino polarization in its rest frame. The formula for the density operator of the polarized neutrino in the limit of vanishing neutrino mass  $m_\nu$  is given by:

$$\begin{aligned} \lim_{m_\nu \rightarrow 0} \Lambda_\nu^{(s)} &= \lim_{m_\nu \rightarrow 0} \frac{1}{2} \left\{ [(q^\mu \gamma_\mu) + m_\nu] [1 + \gamma_5 (S'^\mu \gamma_\mu)] \right\} \\ &= \frac{1}{2} \left\{ (q^\mu \gamma_\mu) [1 + \gamma_5 (\hat{\boldsymbol{\eta}}_\nu \cdot \hat{\mathbf{q}}) + \gamma_5 S'^\perp \cdot \boldsymbol{\gamma}] \right\}, \end{aligned} \quad (\text{A.4})$$

where  $S'^\perp = (0, \boldsymbol{\eta}_\nu^\perp = \hat{\boldsymbol{\eta}}_\nu - (\hat{\boldsymbol{\eta}}_\nu \cdot \hat{\mathbf{q}}) \hat{\mathbf{q}})$ . We see that in spite of the singularity  $m_\nu^{-1}$  in the polarization four-vector  $S'$ , the density operator  $\Lambda_\nu^{(s)}$  including the transverse component of the neutrino spin polarization remains finite [16].

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